Development of Iterative Algorithms for Industrial Tomography

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Abstract *– The limited-angle problem and the divergence of the probing beam are considered as the features of industrial tomography. Modifications of the algebraic algorithms ART and MART are studied under limited-angle conditions. In particular, the non-linear backprojection estimations are suggested as a first guess. A new iterative algorithm based on the Neumann series decomposition is developed for the tomography in divergent beams. Numerical simulations are carried out.*

Keywords: limited-angle tomography, iterative algorithms

1. INTRODUCTION

The investigation of the internal structure of objects with penetrating radiation leads to the tomography problem. The latter has been studied in detail for medical applications [1]. The industrial tomography, however, has its own features, which require the special methods and algorithms to be developed. The present paper considers two aspects of the problem.

The first one is the so-called limited-angle problem. When a part of an engineering construction is tested, a restriction is usually imposed on the viewing angle. In [2] a two-dimensional case has been considered. An extension of this investigation to the case of three-dimensional objects seems to be necessary. Some results on the 3D limited-angle problem have been obtained previously by the authors in [3]. Further development of this study is reported here.

The second point is connected with the geometry of data measurements. The conventional medical tomography equipment allows one to consider the probing beam to be infinitely thin (the ray approximation). But such an assumption is often not possible when one deals with the industrial tomography. The problem of the 3D tomography with finite-aperture beam has been solved with algebraic algorithms in [4]. In this paper another approach is put forward.

Let us consider the X-ray absorption tomography problem. Let some assumptions about the radiation propagation and detection be valid. Then the reconstruction of the three-dimensional distribution of X-ray absorption coefficients from radiation intensity measurements is mathematically equivalent to derivation of function of three variables from its integrals over some spatial domains (in particular, along straight lines for the ray approximation) [1,4,5]. Formally one can write down:

$$Ag = f \,, \qquad (1)$$

where A is a linear projecting operator, g and f are the function to be reconstructed and the projection data, respectively.

The considered features restrict the choice of mathematical tools for inverting Eq.(1). Iterative algorithms (in particular, algebraic ones) seem to be preferable. The purpose of the paper is the development of such algorithms for the real problem statements arising in the industrial applications of tomography.

2. THEORY

2.1. Non-linear backprojection estimations

If there is a lack of the projection data, as a rule, the discrete form of the equation (1) should be considered. Usually, it is a system of linear algebraic equations. Then A is an I ´ J matrix, g Î R^J, f Î R^I.

The approach to solve Eq.(1) often used in technical applications is the evaluation of the unknown absorption distribution by means of the backprojection. The non-linear backprojection estimations are usually not capable of providing the solutions of Eq.(1) [6,7]. For this reason they do not yield the actual value of X-ray absorption coefficients. Nevertheless, they are simple, clear, and effective methods.

Let us consider the scheme of the measurements, which forms a basis for a major part of the X-ray industrial tomography setups. The source of a penetrating radiation is situated at a point ***S*** with spherical coordinates (s,q,j). The radiation having passed through the object under study is registered with a 2D planar detector. The detector front surface is the plane ***D*** defined by the equation *(**r**,**n**)=d*, ***r*** *Î* R^3, ***n*** *Î* S^2. The angular coordinates of ***n*** are *(q+p/2,j-p)*. The distribution of radiation intensity on the detector is often called a *2D projection*. A set of a few projections can be used for the tomography reconstruction.

Assume one has M 2D projections. An averaged backprojection estimation of the unknown function g in a node j can be written down as

$$g_j = \frac{1}{M}\sum_{m=1}^{M} f_{mj} \ , \qquad (2)$$

where f_{mj} is the value of the m-th projection in the point connected with the j-th node. The formula (2) can be generalized to provide different classes of estimations [6]. The *minimum projection method* (MPM) is a simple but fruitful non-linear modification of Eq.(2). It is expressed as

$$g_j = \min(f_{1j}, f_{2j}, ..., f_{Mj}). \qquad (3)$$

Some reconstruction results provided by the estimations (2) and (3) can be found in [7], where elements of building were tested.

2.2. Iterative algebraic algorithms

Iterative algebraic algorithms are used to solve the system of linear equations arising after the discretization of Eq.(1). Two of them, ART and MART, will be considered below.

One of the often-used formula realizing the algorithm ART is the following, [1]:

$$g^{(k+1)} = g^{(k)} + l^{(k)} \frac{f_{i(k)} - (a^{i(k)}, g^{(k)})}{\left\| a^{i(k)} \right\|} a^{i(k)}. \qquad (4)$$

Here $a^{i(k)}$ is the $i(k)$ row of the matrix A, $\lambda^{(k)}$ is the relaxation parameter, $i(k)=[k(mod\ I)+1]$; (· , ·), ‖·‖ are the scalar production and the Euclidean norm in R^J. As was shown in [1], the iterative process (4) converges if $0 < \lambda^{(k)} < 2$ for *"k* and for an arbitrary first guess from the linear span of the rows of the matrix A. Normally, the zero vector is taken as a first guess, if there is no good reasons to use any other one.

The realization of the algorithm MART according to [8] is

$$g_j^{(k+1)} = g_j^{(k)} \left(\frac{f_{i(k)}}{(a^{i(k)}, g^{(k)})} \right)^{l^{(k)} a^{i(k)}} . \qquad (5)$$

Here the index j numbers the elements of the vectors, the other symbols being the same as in (4). The iterative process (5) converges for the first guess being chosen as $g^{(0)}=e^{-1}\mathbf{1}$ (*e* is the base of natural logarithms, **1** is the vector with all components being equal to 1) and under some additional assumptions being satisfied [8].

Some features of the algebraic algorithms were studied in [1,8-14]. The behaviour of ART and MART in the three-dimensional case under limited-angle conditions was considered in [3].

The solution of the ray tomography problem given by such algorithms as ART or MART is known to reveal amplitude oscillations even for the reconstruction from noise-free projection data [1,9]. Therefore, the including of some additional procedures correcting the solution can be useful. In [12] a median filter has been introduced into the iterative process. In [13,14], a generalization of the method has been carried out.

The computational cycle of the steps (4) or (5) when all projection data are processed at once is usually called an *iteration* of the algorithm ART or MART, respectively. (Hence, the iteration is equal to I steps defined with (4) or (5).) Let A^{-1} be an iteration of an algebraic algorithm (ART or MART). Let $\Phi_s^{(n)}$ and $\Phi_f^{(n)}$ be the operators connected with the additional filtration and *a priori* information processing at the *n*-th iteration in the spatial and Fourier domains, respectively. Then the iterative scheme is represented by

$$g^{(n+1)}(x,y,z) = \Phi_s^{(n)} F_3^{-1} \Phi_f^{(n)} F_3 A^{-1} g^{(n)}(x,y,z). \qquad (6)$$

In Eq.(6), F_3 and F_3^{-1} are the operators of the direct and inverse three-dimensional Fourier transforms, respectively. Some forms of operators $\Phi_s^{(n)}$ and $\Phi_f^{(n)}$ have been reported elsewhere [13,14].

It has been shown in [3,13,14] that the using of the general scheme (6) leads to the improvement of reconstruction quality when there takes place a lack of the projection data. In this study these conclusions have been supported with the numerical simulations in which some features of the non-destructive testing have been modelled.

2.3. Influence of the first guess choice for algorithms ART and MART

We propose the estimations (2) and (3) to be used as a first guess for the algebraic algorithms ART and MART in order to improve a reconstruction quality, in particular, in case of limited angular range of projections.

From the theory [1] the algorithm ART provides either the solution of the system (1) having a least norm or the normal one. Therefore, the ART solution if a convergence of an iterative process takes place does not depend on the first guess. The behaviour of the algorithm MART is less studied. From the viewpoint of general theorems of the functional analysis one can expect the MART solution's being also independent. Nevertheless, for numerical implementations it seems to be useful to investigate the dependence on the first guess of the solutions yielded by the algorithms ART and MART. Some reasons for that are listed below.

1. The pure theory deals with the infinite iteration number but in practice the latter is always finite.
2. As a rule, one cannot verify the belonging of a first guess to the manifold ensuring the iterative process convergence.
3. Usually, some additional procedures are involved into the iterative process (see above). Generally speaking, the latter leads to the convergence proof's losing validity.

2.4. Iterative algorithm for wide-beam tomography

Let us suppose that the ray approximation is not valid. Such situation can take place under some conditions for the experimental scheme described in [7]. Then the elements of matrix A become too complicated to be easily determined and the inversion of Eq.(1) with algebraic algorithms needs too much computation time [4]. The other way to solve Eq.(1) is developed below.

Let us represent the operator A as $A=BA_0$, where A_0 corresponds to the ray approximation. From this definition one has $B=AA_0^{-1}$. As far as A_0 describes the ray approximation, many ways of the realization of the operator A_0^{-1} are known. Then one may conclude that the operator B exists. Let us assume that A^{-1} and B^{-1} also exist. Then

$$g = A^{-1} f = (BA_0)^{-1} f = A_0^{-1} B^{-1} f \,. \qquad (7)$$

Using the Neumann series decomposition of the operator B^{-1}, one can write down:

$$g = A_0^{-1} \sum_{k=0}^{\infty} l^k (E - AA_0^{-1})^k f \;, \qquad (8)$$

where E is the unit operator, l is a real parameter. The convergence of the left side of (8) takes place when $|l| \cdot \| E-AA_0^{-1} \| < 1$. If the operator ($E$- AA_0^{-1}) is limited the latter condition can be always satisfied.

3. NUMERICAL SIMULATIONS AND DISCUSSION

3.1. Investigations of ART and MART under limited-angle conditions

A three-dimensional model of the absorption coefficient distribution has been chosen as three cylinders with an internal structure. The cross-sections of the model by the planes X=0 and Z=0 are presented in Figs.1 and 2, respectively.

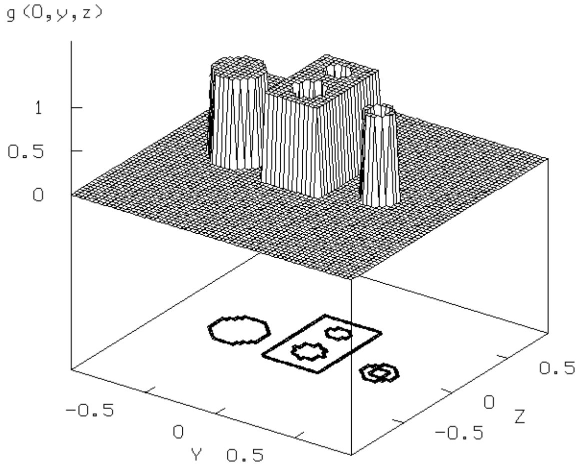

Figure 1: Cross-section of 3D model by plane X=0

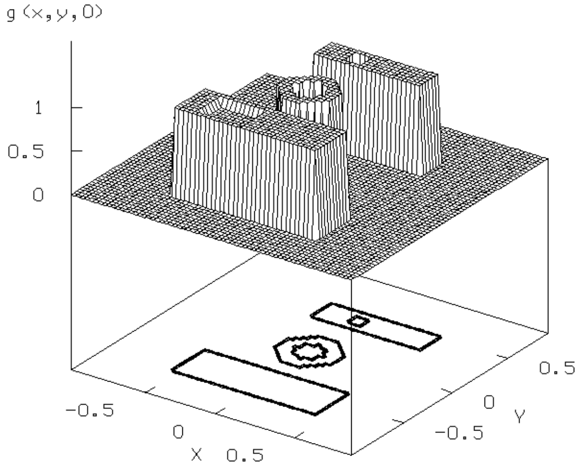

Figure 2: Cross-section of 3D model by plane Z=0

The unit ball in R^3 is considered as a reconstruction domain. The grid of $129\times129\times129$ nodes is determined on the circumscribing cube. The supports of the projections are sampled into 129×129 nodes. The normalized mean root square error Δ is used to evaluate the reconstruction quality.

Let us call the schemes of projection geometry the *equatorial schemes* if the polar angle q =90^o for every projection (see section 2.1). They are of great importance in the 3D tomography, since in this special case the 3D problem reduces to a set of 2D problems.

The summary of the results obtained for the equatorial schemes is as follows. For all modifications of ART and MART, the reconstruction quality decreased unevenly when the azimuthal angular range diminishes. When the range diminishes from 180 to 130-120 degrees, there is no clear tendency to any decrease in the reconstruction quality. Afterwards, the accuracy begins to drop insignificantly. From 60-70 degrees its dramatic decrease is observed. In the narrow angular projection ranges of 20-40 degrees, the combined algorithm cART (see [14]) provides significantly better results than the others. The use of regularizing filters [13,14] improves the reconstruction quality. The adaptive frequency filter [13] surpasses the median one.

Figure 3 illustrates the above conclusions. Here the dependencies of Δ on the azimuthal angular range of projections Dj (the equatorial geometry) are given. The number of projections used for the reconstruction is 13. They are distributed equidistantly over the angular range. Curves 1, 2, 3, and 4 in Fig.3 describe ART, cART, cART with the adaptive frequency filter, and MART, respectively.

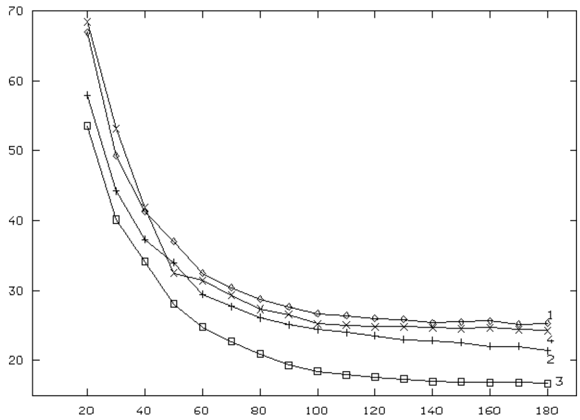

Figure 3: Dependencies of Δ on the angular range of projections Dj, 13 equatorial projections; curve 1 - ART, curve 2 - cART, curve 3 - cART with the adaptive frequency filter, curve 4 - MART

The cross-sections by the plane Z=0 of the solutions yielded by cART with the adaptive frequency filter are given in Figs.4 and 5. In these figures the reconstruction from 13 equatorial projections restricted in the azimuthal angle interval of 30 and 90 degrees, respectively, is presented.

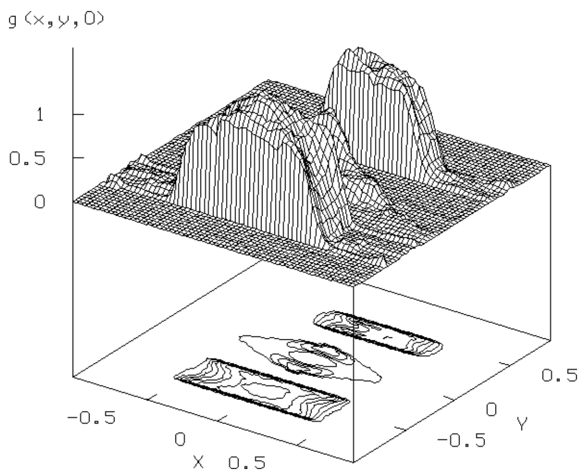

Figure 4: Cross-section of solution by plane Z=0, cART with the adaptive frequency filter, 13 equatorial projections, Dj = 30 degrees, Δ=42.5%

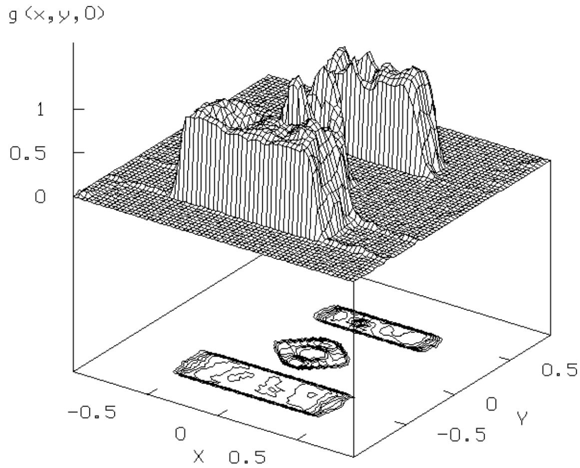

Figure 5: Cross-section of solution by plane Z=0, cART with the adaptive frequency filter, 13 equatorial projections, Dj = 90 degrees, Δ=23.2%

A comparison of the reconstruction qualities provided by the equatorial and non-equatorial schemes with the same projection number has been carried out. The equipment was assumed to collect data in intervals covering 75 degrees along both the azimuthal and polar angles. Different variants of projection location have been considered. The pure equatorial projection geometry with no the variation of the polar angle has been found to provide the worst reconstruction quality. Thus, the advantage of the true 3D tomography over the equatorial one, which can be reduced to the set of 2D problems, has been shown.

3.2. Investigation of the influence of the first guess

To investigate the first guess choice influence on the quality of the reconstruction with the algorithms ART and MART, the same 3D model as in the previous section (see Figs.1, and 2) was used. Both the full geometry and limited-angle one have been considered.

Four different first guesses have been studied.

1. The "standard" first guess, being zero vector for ART and e^{-1} **1** for MART.
2. The estimation obtained with the averaged backprojection, Eq.(2).
3. The estimation obtained with the MPM, Eq.(3).
4. A certain part of 3D-model.

The first guess choice has been shown to have an influence on the solutions yielded by the iterative algebraic algorithms ART and MART. The using of the non-linear estimations as the first guess resulted in a significant improvement of the reconstruction quality. So, in the numerical simulations carried out the normalized mean root square error has been diminished by a factor of 1.2.

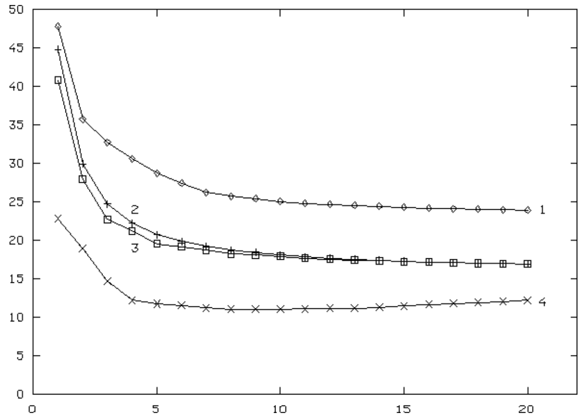

Figure 6: Investigation of the first guess influence, dependencies of Δ on the iteration number *n,* 9 projections of the full equatorial geometry, ART with the median filtration; curve 1 - "standard" first guess, curve 2 - averaged backprojection estimation, curve 3 – MPM estimation, curve 4 - part of the exact model

Some results of the reconstruction one can see in the figures. In Figs.6 and 7, the dependencies of Δ on the iteration number *n* are shown. In the both plots, curves 1 relate to the "standard" first guess, curves 2 - to the averaged backprojection estimation used as the first guess, curves 3 - to the MPM estimation, curve 4 - to the three cylinders with no internal structure. Figure 6 presents the reconstruction by ART and Fig.7 that by MART. Nine projections of the full equatorial geometry have been used for the reconstruction.

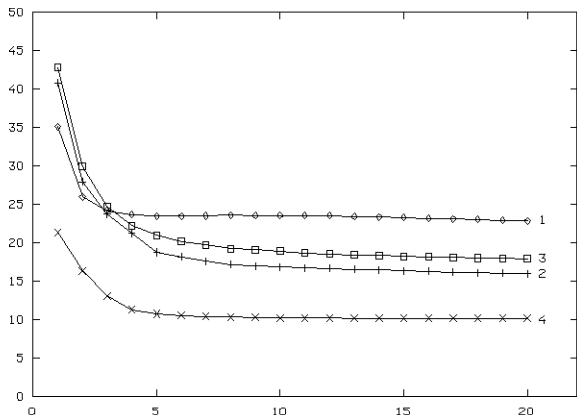

Figure 7: Effect of the first guess influence, dependencies of Δ on the iteration number *n,* 9 projections of the full equatorial geometry, MART with the median filtration; curve 1 - "standard" first guess, curve 2 - averaged backprojection estimation, curve 3 – MPM estimation, curve 4 - part of the exact model

The cross-sections by the plane X=0 of the solutions are given in Figs.8-10. Figure 8 corresponds to the averaged backprojection estimation. Figures 9 and 10 depict the reconstruction with ART, including the median filtration. The "standard" first guess was used for Fig.9 and the averaged backprojection estimation is the first guess for Fig.10.

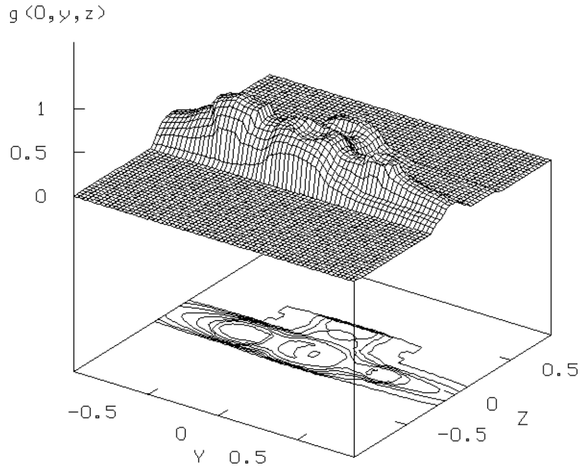

Figure 8: Cross-sections by the plane X=0 of the averaged backprojection estimation, 9 projections of the full equatorial geometry, Δ=76.8%

3.3. Wide-beam tomography algorithm testing

The iterative algorithm (8) has been realized for the two-dimensional tomography problem. The full projection geometry has been considered. The operator A_0^{-1} has been realized with the method of filtered backprojection, using the Shepp-Logan filter [15]. The obtained results seem to be satisfactory.

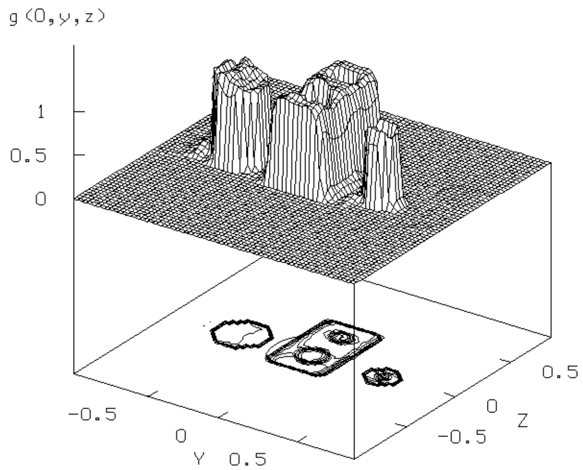

Figure 9: Cross-sections by the plane X=0 of the reconstruction results, 9 projections of the full equatorial geometry, ART with the median filtration, "standard" first guess, Δ=24.5%

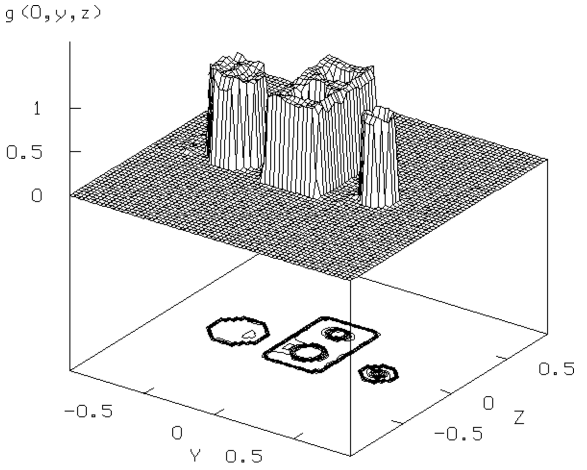

Figure 10: Cross-sections by the plane X=0 of the reconstruction results, 9 projections of the full equatorial geometry, ART with the median filtration, the averaged backprojection estimation is the first guess, Δ=17.4%

The advantage of the suggested algorithm one can see from the Figs.11-13. The two-dimensional model is presented in Fig.11. The reconstructions from 36 1D projections are given in Figs.12 and 13. The averaged width of the probing beam in the reconstruction domain is 6.0 pixel sizes. The reconstruction with the classical Shepp-Logan algorithm is shown in Fig.12 and the reconstruction with the algorithm (8) in Fig.13.

4. CONCLUSION

The performed studies demonstrate that iterative methods are very useful for the industrial applications of tomography.

The three-dimensional limited-angle problem has been solved with the algebraic algorithms ART and MART. The numerical simulations show that it is possible to obtain quite satisfactory results for the angular ranges of projections covering 50 – 70 degrees. The reconstruction quality can be improved by using *a priori* information. In particular, the non-linear backprojection estimations have been suggested as the first guess for the iterative process.

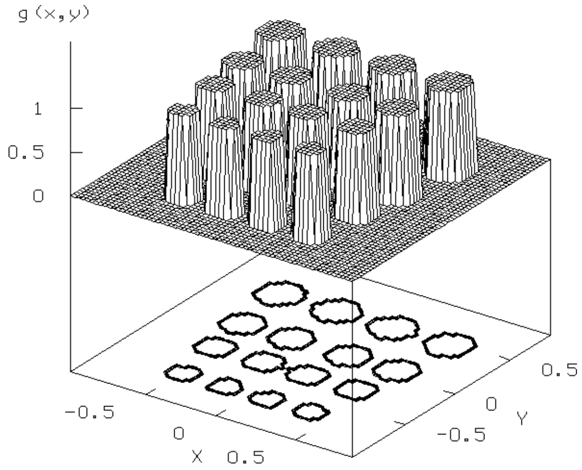

Figure 11: Two-dimensional model

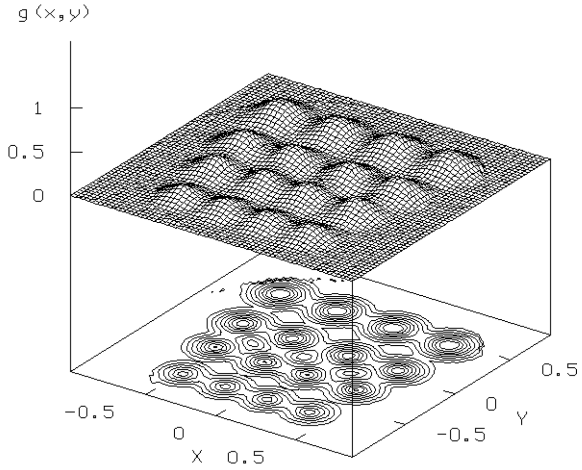

Figure 12: Two-dimensional reconstruction, 36 1D projections, width of the probing beam is 6.0 pixels, classical Shepp-Logan algorithm, Δ=92.4%

A new iterative algorithm has been developed for the statements where the ray tomography approximation is not valid. It proved to be quite effective for some special tasks arising in non-destructive testing.

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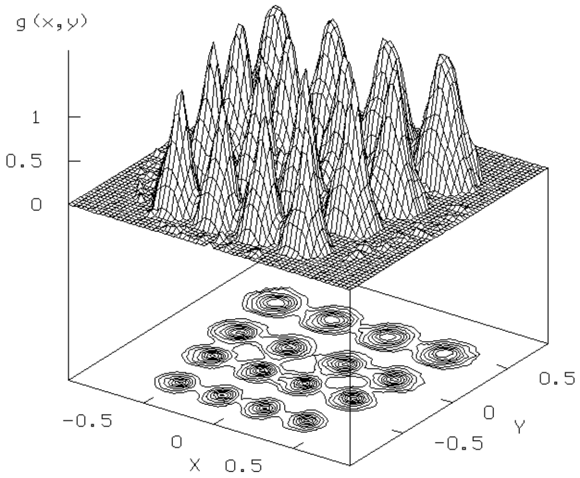

Figure 13: Two-dimensional reconstruction, 36 1D projections, wide of the probing beam is 6.0 pixels, suggested iterative algorithm, Δ=45.3%

REFERENCES

[1] G.T. Herman, "*Image Reconstruction from Projections: The Fundamentals of Computerized Tomography*". New York: Academic Press, 1980.

[2] D. Verhoeven, "*Limited-data computed tomography algorithms for the physical sciences*", *Applied Optics*, 1993, **32**(20), pp. 3736-3754.

[3] G.G. Levin, G.N. Vishnyakov, C.S. Zakarian, A.V. Likhachov, V.V. Pickalov, G.I. Kozinets, J.K. Novoderzhkina and E.A. Streletskaya, "*Three-dimensional limited-angle microtomography of blood cells: experimental results*", *SPIE Proceedings*, 1998, **3261**, pp. 159-164.

[4] A.V. Likhachov and V.V. Pickalov, "*Three-dimensional tomography with finite aperture beam*", *Nucl. Instr. and Meth.*, 1998, **405**, pp. 506-510.

[5] H.H. Barrett and W. Swindell, "*Radiological Imaging*" vol. 1. New York: Academic Press, 1981.

[6] A.V. Baranov, "*A variational approach to non-linear backprojection*", "*Computerized Tomography. Proc. Fourth Intern. Sympos. Novosibirsk, Russia*", Utrecht: VSP, 1995, pp. 82-97.

[7] U. Ewert, A.V. Baranov and K. Borchardt, "*Cross-sectional imaging of building elements by new non-linear tomosynthesis techniques using imaging plates and Co[60] radiation*", *NDT & E International,* 1997, **30**(4), pp. 243-248.

[8] Y. Censor, "*Finite series-expansion reconstruction methods*", *Proc. IEEE*, 1983, **71**, pp. 409-419.

[9] A.V. Likhachov and V.V. Pickalov, "*A modification of ART method for cone-beam tomography of high space resolution*", "*Computerized Tomography. Proc. Fourth Intern. Sympos. Novosibirsk, Russia"*, Utrecht: VSP, 1995, pp. 309-317.

[10] A.V. Likhachov and V.V. Pickalov, "*Subpixel resolution in 3D cone-beam microtomography*", *Nucl. Instr. and Meth.*, 1995, **359**, pp. 370-375.

[11] G.T. Herman, "*Image reconstruction from projections*", *Real-Time Imaging*, 1995, **1**, pp. 3-18.

[12] R.M. Rangayyan and R. Gordon, "*Streak preventive image reconstruction with ART and adaptive filtering*", *IEEE Trans. Med. Imag.*, 1982, **1**, pp. 173-178.

[13] A.V. Likhachov and V.V. Pickalov, "*Frequency filtration in the algebraic algorithms of 3D-tomography*", "*Proc. 1995 Intern. Meeting on Fully 3D Image Reconstruction in Radiology and Nuclear Medicine*". France: LETI, pp. 103-107.

[14] A.V. Likhachov and V.V. Pickalov, "*Frequency filtration in algebraic algorithms of three-dimensional tomography*", *Optoelectronics, Instrumentation and Data Processing, 1995,* **4**, pp. 83-89.

[15] L.A. Shepp and B.F. Logan, "*The Fourier reconstruction of a head section*", *IEEE Trans. NS*, 1974, **21**, pp. 21-43.